Caringbah High School

Year 12 2019
Mathematics
HSC Course
Assessment Task 4

General Instructions
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11-16, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for partial or incomplete answers

Total marks – 100

Section I **10 marks**
Attempt Questions 1-10
Mark your answers on the answer sheet provided. You may detach the sheet and write your name on it.

Section II **90 marks**
Attempt Questions 11-16
Write your answers in the answer booklets provided. Ensure your name or student number is clearly visible.

Name: ______________________ **Class:** ___________

Marker's Use Only								
Section I	**Section II**						**Total**	
Q 1-10	**Q11**	**Q12**	**Q13**	**Q14**	**Q15**	**Q16**		
/10	/15	/15	/15	/15	/15	/15	/100	%

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–5

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1. What is the value of $\log_2 3$, correct to 4 significant figures?

   (A) 1.584
   (B) 1.585
   (C) 0.6309
   (D) 0.6310

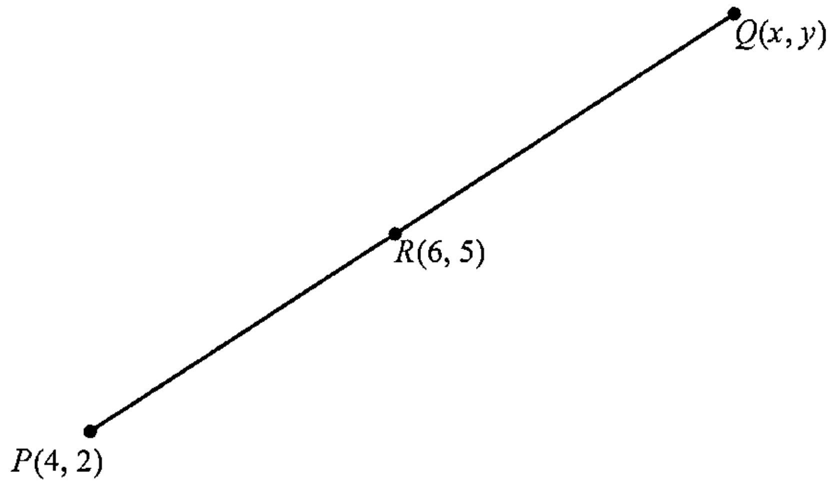
2. The primitive function of $\sec^2 2x$ is...

   (A) $\tan x + c$
   (B) $\tan 2x + c$
   (C) $\frac{1}{2}\tan x + c$
   (D) $\frac{1}{2}\tan 2x + c$

3. How many solutions does the equation $(\cos x + 1)(2\sin x - 1) = 0$ for $0 \leq x \leq 2\pi$ have?

   (A) 2
   (B) 3
   (C) 4
   (D) 5

4. The point $R(6, 5)$ is the midpoint of the interval PQ, where P has the coordinates $(4, 2)$. What is the coordinate of Q?



(A) $(9, 9)$

(B) $(7, 8)$

(C) $(8, 8)$

(D) $(8, 9)$

5. What are the domain and range of $f(x) = \sqrt{4 - x^2}$?

(A) Domain: $-2 \le x \le 2$ Range: $0 \le y \le 2$

(B) Domain: $-2 \le x \le 2$ Range: $-2 \le y \le 2$

(C) Domain: $0 \le x \le 2$ Range: $-4 \le y \le 4$

(D) Domain: $0 \le x \le 2$ Range: $0 \le y \le 4$

6. A particle moves so that its velocity function at time t seconds is given by:

$$v = 2e^{-t}(1 - t)$$

At what time is the acceleration zero?

(A) $t = 0$

(B) $t = 1$

(C) $t = 2$

(D) $t = 3$

7. Find the perimeter (P) of the sector of a circle with radius 10 cm and an angle of $\frac{\pi}{3}$ subtended at the centre.

(A) $P = 0.5 \times 100 \times \left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$ cm

(B) $P = \left(0.5 \times 100 \times \frac{\pi}{3}\right)$ cm

(C) $P = \left(20 + \frac{\pi}{3}\right)$ cm

(D) $P = \left(20 + \frac{10\pi}{3}\right)$ cm

8. Which of the following is equal to $\frac{1}{3\sqrt{5}+\sqrt{2}}$?

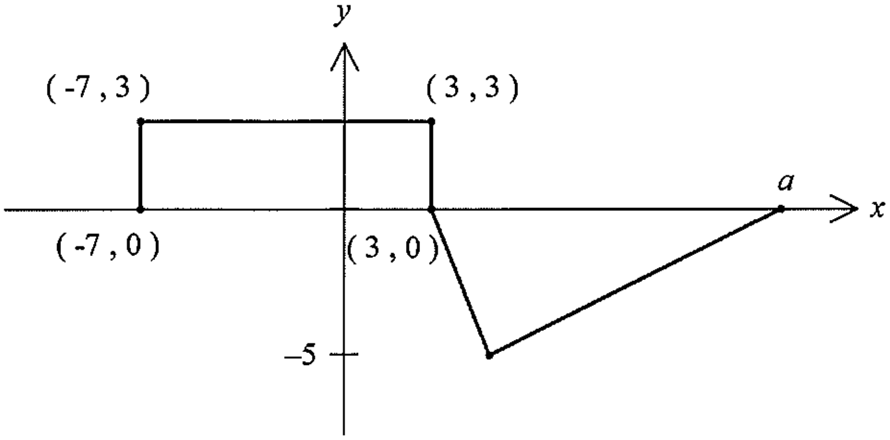
(A) $\frac{3\sqrt{5} - \sqrt{2}}{13}$

(B) $\frac{3\sqrt{5} + 2}{13}$

(C) $\frac{3\sqrt{5} - \sqrt{2}}{43}$

(D) $\frac{3\sqrt{5} + \sqrt{2}}{43}$

9. The diagram shows the graph of $y = f(x)$.



Use the graph to determine the value of a which satisfies the condition

$$\int_{-7}^{a} f(x)dx = 0.$$

(A) 9

(B) 12

(C) 13

(D) 15

10. The solution of the equation $\ln(x+2) - \ln x = \ln 4$ is

(A) $x = \frac{2}{3}$

(B) $x = \frac{2}{5}$

(C) $x = \frac{3}{2}$

(D) $x = \frac{5}{2}$

End of Section I- Multiple Choice Questions

Section II

60 marks
Attempt Questions 11–16
Allow about 2 hours 45 minutes for this section

Answer each question in a SEPARATE writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Calculate correct to one decimal place, the value of **1**

$\sqrt{\frac{3xy}{z}}$ when $x = 4.2$, $y = 6.8$ and $z = 4.4$

(b) Simplify: $\dfrac{x - x^{-2}}{1 + x^{-2}}$ **2**

(c) Differentiate:

(i) $\dfrac{2}{5x^3}$ **1**

(ii) $e^x \sin x$ **2**

(iii) $(e^{3x} - 5)^4$ **2**

(d) A parabola has an equation given by: $y = \frac{1}{2}(x^2 - 6x + 19)$

(i) Express the above equation in the form $(x - h)^2 = 4a(y - k)$. **2**

(ii) Find the coordinates of the vertex and focus of the parabola. **2**

(iii) Find the equation of the directrix of the parabola. **1**

(iv) Sketch the locus of P, indicating all the above features. **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Find the primitive of $\cos(2x + 1)$. **1**

(b) If α and β are the roots of the equation $2x^2 - 3x + 4 = 0$, find the value of $\alpha^2 + \beta^2$. **3**

(c) Find, in general form, the equation of the tangent to the curve $y = x\ln x$ at the $x -$ intercept. **3**

(d) Find $\int_0^1 \frac{3x}{x^2 + 1}\, dx$, in exact form. **3**

(e) Solve and graph the solution for $|1 - 3x| \geq 2$. **3**

(f) For what values of k will the equation $x^2 - kx + 9 = 0$ have real and different roots? **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) Find

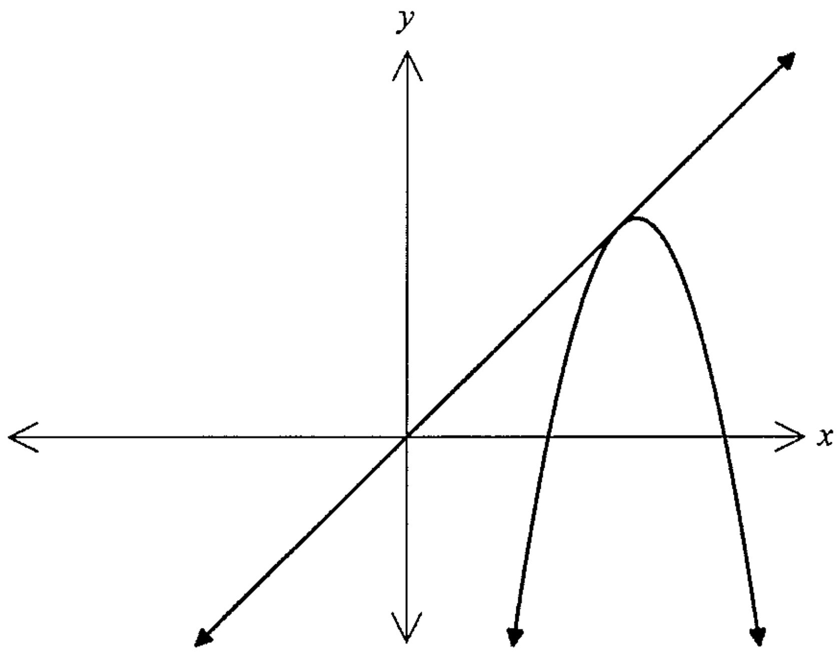
(i) $\int e^{5x+1} dx$ **1**

(ii) $\int \cot x \, dx$ **1**

(iii) $\int (x^2 - 1)^2 \, dx$ **2**

(b) Evaluate $\int_0^3 \sqrt{5x+1} \; dx.$ **3**

(c) The parabola $y = -x^2 + 13x - 36$, passes through the point $P(6, 6)$.



(i) Show the equation of the tangent to the parabola at the point $P(6,6)$ is $y = x$. **2**

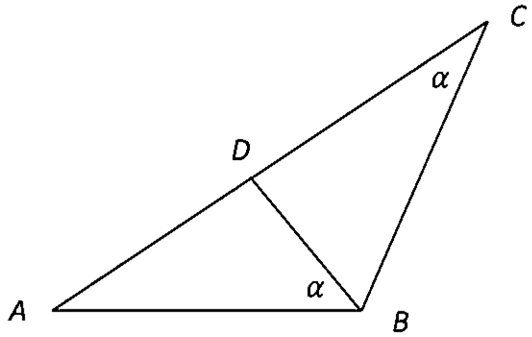
(ii) Find the area bounded by the parabola, the tangent and the $x -$ axis. **4**

(d) Solve $2^{2x} - 5 \times 2^x + 4 = 0$ **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a)



(i) Prove that ΔABC and ΔADB are similar. **2**

(ii) If $AD = 4$ cm and $DC = 12$ cm, find the length of AB. **1**

(b) Let $f(x) = \frac{1}{3}x^3 + x^2 - 3x + 5$.

(i) Find the stationary points and determine their nature. **4**

(ii) Find any inflexion points. **2**

(iii) Sketch the graph of $f(x)$. **1**

(iv) For what value of x is $f(x)$ concave up? **1**

(c) The mass, M, in grams of radioactive substance is expressed as $M = 195e^{-kt}$ where k is a positive constant and t the time in days.
The mass of the substance halved in 6 days.

(i) Find the value of k, correct to 4 decimal places. **2**

(ii) At what rate is the mass decaying after 15 days. Answer correct to one decimal place. **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) Amelia is saving for a holiday. In the first month she saves \$30, in the second month her savings are \$5 more than the month before.

(i) How much will she save in the 21st month? **1**

(ii) How much money will she have saved in total by the 21st month? **2**

(iii) Amelia needs \$2100 to pay for her plane ticket. How long will it take her to save this amount? **2**

(b) Calculate the volume of a solid of revolution if $y = \frac{1}{\sqrt{x}}$ is rotated about the $y-$axis from $y = 1$ to $y = 2$. **2**

(c) A particle is travelling in a straight line, starting from the origin, such that

$$\frac{d^2x}{dt^2} = \frac{8}{(t+1)^2}$$

where x is displacement in metres and t is time in seconds.

(i) Explain why the acceleration is always positive. **1**

(ii) Find the expression for velocity of the particle. **2**

(iii) Find the distance covered between $t = 2$ and $t = 5$ seconds. **3**

(d) Solve $3\tan 2x = \sqrt{3}$ for $0 \leq x \leq 2\pi$ **2**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a) Sam borrowed \$18 000 to buy a new car. He is charged interest at 12% p.a. compounded monthly, on the balance owing. The loan is to be repaid in equal monthly instalments over 5 years.
Let A_n be the amount owing after the *nth* monthly repayments M has been made.

(i) Write an expression for the amount owing after one month, A_1. **1**

(ii) Show that $A_n = 18000(1.01)^n - 100M(1.01^n - 1)$ **3**

(iii) Calculate Sam's monthly instalment. **1**

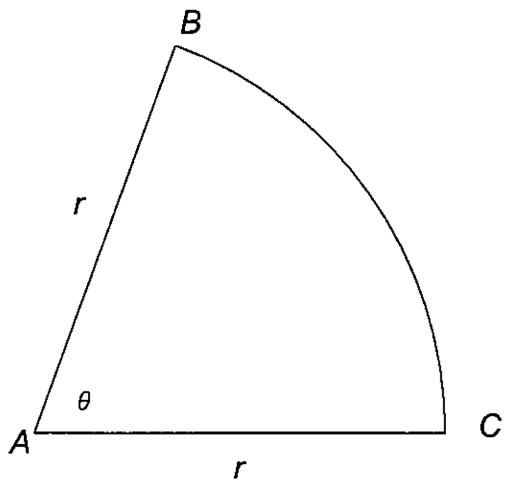
(b) If $2x^2 - 3x + 3 \equiv A(x-1)^2 + B(x-1) + C$, find the value of A, B and C. **2**

(c) Use Simpson's Rule with 3 function values to estimate **2**

$$\int_0^{\frac{\pi}{4}} \sec x \, dx$$

Answer correct to two decimal places.

(d) AB and AC are radii of the length r metres of a circle with centre A. The arc BC of the circle subtends an angle of θ at A. The perimeter of figure ABC is 8 m.



(i) Show that the area A m^2 of the sector ABC is given by $A = \frac{32\theta}{(2+\theta)^2}$. **2**

(ii) Hence, find the maximum area of the sector. **4**

End of Paper!

Yr 12 Trial Mathematics 2019 Solutions

Section I

Q1	B	Q6	C
Q2	D	Q7	D
Q3	B	Q8	C
Q4	C	Q9	D
Q5	A	Q10	A

Q1 $\log_2 3 = \dfrac{\log_e 3}{\log_e 2}$

$= 1.585$ (B)

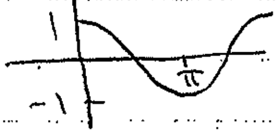
Q2 $\int \sec^2 2x \, dx = \frac{1}{2}\tan 2x + c$ (D)

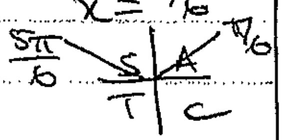
Q3 $(\cos x + 1)(2\sin x - 1) = 0$ $\quad 0 \le x \le 2\pi$

$\cos x = -1 \qquad 2\sin x = 1$

$x = \pi \qquad \sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$





$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \pi$

$\therefore$ 3 solutions (B)

Q4 $\dfrac{4+x}{2} = 6, \quad \dfrac{y+2}{2} = 5$

$4 + x = 12 \qquad y + 2 = 10$

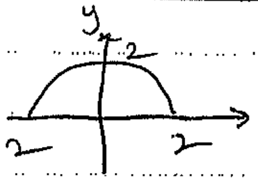
$x = 8 \qquad y = 8.$

OR

by inspection.

$(8, 8)$ (C)

Q5 $f(x) = \sqrt{4 - x^2}$



D: $-2 \le x \le 2$

R: $0 \le y \le 2$ (A)

Q6 $v = 2e^{-t}(1-t)$

$a = 2e^{-t}(-1) + (1-t)(-2e^{-t})$

$= -2e^{-t} - 2e^{-t} + 2te^{-t}$

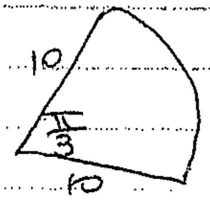
$= -4e^{-t} + 2te^{-t}$

$= 2e^{-t}(-2 + t)$

$2e^{-t}(-2+t) = 0$

$t = 2$ (C)

Q7 $P = r\theta + 2r$



$= 10 \times \frac{\pi}{3} + 2 \times 10$

$= \left(\frac{10\pi}{3} + 20\right)$ cm (D)

Q8 $\dfrac{1}{3\sqrt{5}+\sqrt{2}} \times \dfrac{3\sqrt{5}-\sqrt{2}}{3\sqrt{5}-\sqrt{2}}$

$= \dfrac{3\sqrt{5}-\sqrt{2}}{45 - 2}$

$= \dfrac{3\sqrt{5}-\sqrt{2}}{43}$ (C)

Q9 $A_{rectangle} = 3 \times 10 = 30u^2$

$A_{triangle} = \frac{1}{2} \times b \times 5$

$= \frac{5b}{2}$

$A_1 = A_2$, for $\int_{-7}^{a} f(x)\,dx = 0$.

$\therefore \frac{5b}{2} = 30$

$5b = 60$

$b = 12$

$\therefore a = b + 3$

$= 12 + 3$

$\therefore a = 15$ (D)

Q10 $\ln(x+2)-\ln x=4$

$$\frac{x+2}{x}=4$$

$$x+2=4x$$

$$3x=2$$

$$x=\frac{2}{3}\ (A)$$

Section II

Q11

(a) $\sqrt{\frac{3xy}{z}}=\sqrt{\frac{3\times 4.2\times 6.8}{4.4}}$

$$=4.4$$

(b) $\frac{x-x^{-2}}{1+x^{-2}}=\frac{x-\frac{1}{x^2}}{1+\frac{1}{x^2}}$

$$=\frac{x^3-1}{x^2}\div\frac{x^2+1}{x^2}$$

$$=\frac{x^3-1}{x^2}\times\frac{x^2}{x^2+1}$$

$$=\frac{x^3-1}{x^2+1}$$

(c) (i) $\frac{d}{dx}\left(\frac{2}{5x^3}\right)=\frac{d}{dx}\left(\frac{2}{5}x^{-3}\right)$

$$=\frac{-6}{5}x^{-4}$$

$$=\frac{-6}{5x^4}$$

(ii) $\frac{d}{dx}(e^x\sin x)$

$$=e^x\cos x+\sin x\, e^x$$

$$=e^x(\cos x+\sin x)$$

(iii) $\frac{d}{dx}(e^{3x}-5)^4$

$$=4(e^{3x}-5)^3(3e^{3x})$$

$$=12e^{3x}(e^{3x}-5)^3$$

(d)(i) $y=\frac{1}{2}(x^2-6x+19)$

$$2y=x^2-6x+19$$

$$2y=x^2-6x+9+10$$

$$2y-10=(x-3)^2$$

$$2(y-5)=(x-3)^2$$

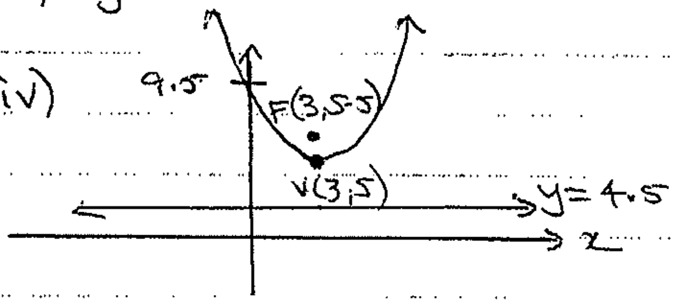
$$\therefore (x-3)^2=2(y-5)$$

(ii) $a=1/2$

$\therefore V(3,5)\quad F(3,5.5)$

(iii) $y=4.5$

(iv)



Q12

(a) $\int\cos(2x+1)\,dx$

$$=\frac{1}{2}\sin(2x+1)+c$$

(b) $2x^2-3x+4=0$

$\alpha+\beta=3/2,\quad \alpha\beta=2$

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$

$$=(3/2)^2-2(2)$$

$$=-1\,3/4$$

Q12 cont.

(c) $y = x\ln x$

$y' = x\left(\frac{1}{x}\right) + \ln x\,(1)$

$= 1 + \ln x$.

x-int, $y = 0$.

$x\ln x = 0$.

$x = 0, 1$

The only tangent that can exist is at $(1, 0)$

when $x = 1$, $y' = 1 + \ln 1$

$\therefore m = 1$

$y - y_1 = m(x - x_1)$

$y - 0 = 1(x - 1)$

$y = x - 1$

$x - y - 1 = 0$.

(d) $\int_0^1 \frac{3x}{x^2+1}\,dx$

$= \frac{3}{2}\int_0^1 \frac{2x}{x^2+1}\,dx$.

$= \frac{3}{2}\left[\ln(x^2+1)\right]_0^1$

$= \frac{3}{2}\left[\ln 2 - \ln 1\right]$

$= \frac{3}{2}\left[\ln 2 - 0\right]$

$= \frac{3}{2}\ln 2$

(e) $|1 - 3x| \geq 2$

$1 - 3x = 2$ $\qquad$ $1 - 3x = -2$

$-3x = 1$ $\qquad$ $-3x = -3$

$x = -\frac{1}{3}$ $\qquad$ $x = 1$

test

$\therefore x \leq -\frac{1}{3},\ x \geq 1$

(f) $x^2 - kx + 9 = 0$.

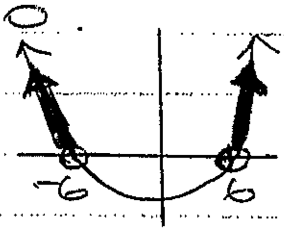
For real & different roots:

$\Delta > 0$

$(-k)^2 - 4(1)(9) > 0$.

$k^2 - 36 > 0$.

$k^2 > 36$



$\therefore k < -6,\ k > 6$.

Q13

(a) (i) $\int e^{5x+1}\,dx$

$= \frac{1}{5}e^{5x+1} + C$

(ii) $\int \cot x\,dx$

$= \int \frac{\cos x}{\sin x}\,dx$

$= \ln \sin x + C$.

(iii) $\int (x^2 - 1)^2\,dx$

$= \int (x^4 - 2x^2 + 1)\,dx$

$= \frac{x^5}{5} - \frac{2x^3}{3} + x + C$

Q13 cont

(b) $\int_0^3 \sqrt{5x+1}\, dx$

$= \int_0^3 (5x+1)^{1/2}\, dx$

$= \left[\frac{2(5x+1)^{3/2}}{3\times 5}\right]_0^3$

$= \frac{2}{15}\left[(5(3)+1)^{3/2} - 1\right]$

$= \frac{2}{15}\left[16^{3/2} - 1\right]$

$= \frac{2}{15}\left[64 - 1\right]$

$= \frac{2}{15} \times 63$

$= \frac{42}{15}$

$= 8\,^2/_5$

(c) (i) $y = -x^2 + 13x - 36 \quad P(6,6)$

$y' = -2x + 13$

sub $x = 6$, $y' = -2(6) + 13$

$= -12 + 13$

$\therefore m = 1$

$y - y_1 = m(x - x_1)$

$y - 6 = 1(x - 6)$

$y = x$

(ii) For $y = -x^2 + 13x - 36$

x-int, $y = 0$.

$x^2 - 13x + 36 = 0$

$(x-9)(x-4) = 0$

$x = 9, 4$.

$A = \frac{1}{2}bh - \int_4^6 \text{parabola}$

$= \frac{1}{2}\times 6 \times 6 - \int_4^6 (-x^2 + 13x - 36)\, dx$

$= 18 - \left[\frac{-x^3}{3} + \frac{13x^2}{2} - 36x\right]_4^6$

$= 18 - \left[\left(\frac{-6^3}{3} + \frac{13(6)^2}{2} - 36(6)\right) - \left(-\frac{4^3}{3} + \frac{13(4)^2}{2} - 36(4)\right)\right]$

$= 18 - \left[-54 - \left(\frac{-184}{3}\right)\right]$

$= 18 + 54 - \frac{184}{3}$

$= \frac{32}{3}$

$= 10\frac{2}{3}u^2$

(d) $2^{2x} - 5\times 2^x + 4 = 0$.

$(2^x)^2 - 5\times 2^x + 4 = 0$

let $u = 2^x$

$u^2 - 5u + 4 = 0$.

$(u-4)(u-1) = 0$.

$u = 4, 1$

But $u = 2^x$

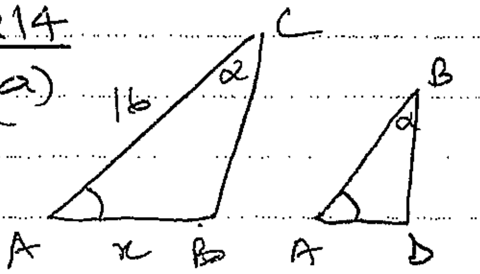
$2^x = 4 \qquad 2^x = 1$

$2^x = 2^2 \qquad 2^x = 2^0$

$\therefore x = 0, 2$

Q14

(a)



(i) In $\triangle ABC$ & $\triangle ADB$,

$\angle ACB = \angle ABD = \alpha$ (given)

$\angle CAB = \angle BAD$ (common $\angle$)

$\therefore \triangle ABC \parallel\!\mid \triangle ADB$ (equiangular)

(ii) $\frac{AB}{AD} = \frac{AC}{AB}$

$\frac{x}{4} = \frac{16}{x}$

$x^2 = 64$

$x = 8$ (as $x > 0$)

b) (i) $f(x) = \frac{1}{3}x^3 + x^2 - 3x + 5$

$f'(x) = x^2 + 2x - 3$

$f'(x) = 0$, stat. pt.

$x^2 + 2x - 3 = 0$.

$(x+3)(x-1) = 0$

$x = -3, 1$

$f(-3) = 14$, $f(1) = 10/3$.

$f''(x) = 2x + 2$

$f''(-3) = 2(-3) + 2$

$= -4 < 0$, max

$f''(1) = 2(1) + 2$

$= 4 > 0$ min

$\therefore (-3, 14)$ is a max t.p &

$(1, 10/3)$ is a min t.p.

(ii) $f''(x) = 2x + 2$

$f''(x) = 0$, inflexion pt.

$2x + 2 = 0$

$2x = -2$

$x = -1$

$f(-1) = 26/3$.

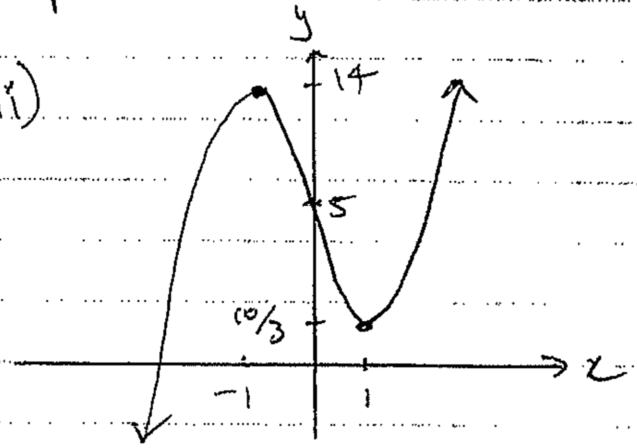
$\therefore (-1, 26/3)$ is a possible inflexion point.

x	-2	-1	0
$f''(x)$	-2	0	2

$\therefore$ change in concavity

$\therefore (-1, 26/3)$ is an inflexion point.

(iii)



(iv) $x > -1$

(c) (i) $M = 195e^{-kt}$

when $t = 0$, $M = 195$

$97.5 = 195e^{-6k}$

$0.5 = e^{-6k}$

$\ln(0.5) = \ln e^{-6k}$

$-6k = \ln(0.5)$

$k = -\frac{\ln(0.5)}{6} = 0.1155$

Q14 cont.

(c)(ii) $\frac{dM}{dt} = -195ke^{-kt}$

$= -195(0.1155)e^{-0.1155(15)}$

$= -3.98...$

$\doteqdot -4.0$ g/day

Q15

(a)(i) 30, 35, 40, ...

$T_{21} = 30 + 20 \times 5$

$= \$130$

(ii) $S_{21} = \frac{21}{2}(30 + 130)$

$= \$1680$

(iii) $2100 = \frac{n}{2}[2(30) + (n-1)5]$

$4200 = n[60 + 5n - 5]$

$4200 = n[5n + 55]$

$4200 = 5n^2 + 55n$

$5n^2 + 55n - 4200 = 0$

$5(n^2 + 11n - 840) = 0$

$n^2 + 11n - 840 = 0$

$(n+35)(n-24) = 0$

$n = -35, 24$

$\therefore n = 24$ since $n > 0$.

$\therefore$ 24 months to pay \$2100

(b) $y = \frac{1}{\sqrt{x}}$, $y = 1$, $y = 2$

$y^2 = \frac{1}{x}$

$x = \frac{1}{y^2}$

$x^2 = \frac{1}{y^4}$

$x^2 = y^{-4}$

$V = \pi\int_a^b x^2\,dy$

$= \pi\int_1^2 y^{-4}\,dy$

$= \pi\left[\frac{y^{-3}}{-3}\right]_1^2$

$= -\frac{\pi}{3}\left[\frac{1}{y^3}\right]_1^2$

$= -\frac{\pi}{3}\left[\frac{1}{2^3} - \frac{1}{1^3}\right]$

$= -\frac{\pi}{3}\left[\frac{1}{8} - 1\right]$

$= -\frac{\pi}{3}\left[-\frac{7}{8}\right]$

$= \frac{7\pi}{24}\,u^3$

(c)(i) $\frac{d^2x}{dt^2} = \frac{8}{(t+1)^2}$

Since both numerator & denominator are both positive, then $\frac{d^2x}{dt^2} > 0$.

Q15 cont.

(c) (ii) $\frac{dx}{dt} = 8\int (t+1)^{-2}\,dt$

$= 8\left[\frac{(t+1)^{-1}}{-1(1)}\right] + c$

$\frac{dx}{dt} = \frac{-8}{t+1} + c$

when $t=0$, $\frac{dx}{dt}=0$

$\frac{-8}{0+1} + c = 0$

$-8 + c = 0$

$c = 8$

$\therefore \frac{dx}{dt} = -\frac{8}{t+1} + 8$

(iii) $x = \int_2^5 \left(\frac{-8}{t+1} + 8\right) dt$

$= -8\int_2^5 \frac{1}{t+1}\,dt + \int_2^5 8\,dt$

$= -8\left[\ln(t+1)\right]_2^5 + \left[8t\right]_2^5$

$= -8\left[\ln 6 - \ln 3\right] + \left[8(5) - 8(2)\right]$

$= -8\left[\ln 6 - \ln 3\right] + (40 - 16)$

$= -8\ln 2 + 24$

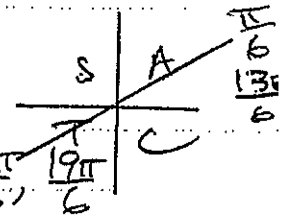
$= 24 - 8\ln 2$

(d) $3\tan 2x = \sqrt{3}$ $\quad 0 \le x \le 2\pi$

$\tan 2x = \frac{\sqrt{3}}{3}$ $\quad 0 \le 2x \le 4\pi$

$2x = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$= \frac{\pi}{6}$



$\therefore 2x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}$

$x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$.

Q16.

(a) \$18000, 12% p.a, 5yrs

= 1% /month, 60 months

(i) $A_1 = 18000(1.01) - M$

(ii) $A_2 = A_1(1.01) - M$

$= [18000(1.01) - M](1.01) - M$

$= 18000(1.01)^2 - M(1.01) - M$

$= 18000(1.01)^2 - M(1.01 + 1)$

$A_3 = A_2(1.01) - M$

$= [18000(1.01)^2 - M(1.01+1)](1.01) - M$

$= 18000(1.01)^3 - M(1.01^2 + 1.01) - M$

$= 18000(1.01)^3 - M(1 + 1.01 + 1.01^2)$

$A_n = 18000(1.01)^n - M(1 + 1.01 + 1.01^2 + \ldots + 1.01^{n-1})$

$= 18000(1.01)^n - M\left[\frac{1(1.01^n - 1)}{0.01}\right]$

$= 18000(1.01)^n - 100M(1.01^n - 1)$

Q16 cont.

(a)(iii) $A_{60} = 18000(1.01)^{60} - 100M(1.01^{60} - 1)$

But $A_{60} = 0$

$18000(1.01)^{60} - 100M(1.01^{60} - 1) = 0$

$100M(1.01^{60} - 1) = 18000(1.01)^{60}$

$M = \frac{18000(1.01)^{60}}{100(1.01^{60} - 1)}$

$\therefore M = \$400.40$

(b) $2x^2 - 3x + 3 \equiv A(x-1)^2 + B(x-1) + C$

RHS:

$A(x-1)^2 + B(x-1) + C$

$= A(x^2 - 2x + 1) + Bx - B + C$

$= Ax^2 - 2Ax + A + Bx - B + C$

$= Ax^2 + (-2A + B)x + A - B + C.$

Equating LHS & RHS:

$A = 2$, $-2A + B = -3$, $A - B + C = 3$

$-2(2) + B = -3$, $2 - 1 + C = 3$

$-4 + B = -3$, $1 + C = 3$

$B = 1$, $C = 2.$

$\therefore A = 2, B = 1, C = 2$

(c) $\int_0^{\pi/4} \sec x \, dx.$

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
$\sec x$	1	$\sec\frac{\pi}{8}$	$\sqrt{2}$

$A \doteqdot \frac{\pi/8}{3}\left[1 + 4 \times \sec\frac{\pi}{8} + \sqrt{2}\right]$

$= 0.88$

(d)(i) $P = 2r + r\theta$

$8 = 2r + r\theta$

$8 = r(2 + \theta)$

$r = \frac{8}{2+\theta}$ ... ①

$A = \frac{1}{2}r^2\theta$ ... ②

sub ① into ②

$A = \frac{1}{2}\left(\frac{8}{2+\theta}\right)^2 \times \theta$

$= \frac{1}{2} \times \frac{64}{(2+\theta)^2} \times \theta$

$= \frac{32\theta}{(2+\theta)^2}$

(ii) $\frac{dA}{d\theta} = \frac{(2+\theta)^2 \times 32 - 32\theta[2(2+\theta)(1)]}{(2+\theta)^4}$

$= \frac{32(2+\theta)^2 - 64\theta(2+\theta)}{(2+\theta)^4}$

$= \frac{32(2+\theta)[2 + \theta - 2\theta]}{(2+\theta)^4}$

$= \frac{32[2-\theta]}{(2+\theta)^3}$

For max area, $\frac{dA}{d\theta} = 0$.

$\frac{32(2-\theta)}{(2+\theta)^3} = 0$, $\theta \neq -2$.

$32(2-\theta) = 0.$

$\theta = 2$

To check nature:

θ	0	2	3
$\frac{dA}{d\theta}$	8	0	$-\frac{32}{125}$
	/	—	\

$\therefore \theta = 2$ is a max.

$\therefore$ Max. area $= \frac{32(2)}{(2+2)^2} = 4\,m^2$